DAV BR PUBLIC SCHOOL, BINA
HALF YEARLY EXAMINATION (SAMPLE PAPER)(2024-25)
CLASS: XII
SUBJECT: APPLIED MATHEMATICS

TIME: 3:00 HOURS **MM:80**

Read the following instructions very carefully and strictly follow them:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. Use of calculators is not allowed.

Section A

1. -8(mod 5) 1
   a) 1 b) 2 c) 3 d) 4

2. In what ratio must a grocer mix two types of pulses worth ₹ 85 per kg ₹ 100per kg respectively so as to get a mixture worth ₹92 per kg 1
   a) 2:1 b) 1:2 c) 8:7 d) 4:3

3. A man rows 60 km upstream and 40 km downstream in 10 hours each time. What is the speed of the boat? 1
   a) 1km/h b) 8 km/h c) 5km/h d) 3km/h

4. Rakesh and suresh invest in a business in the ratio 4:5. If 10% of the profit goes for charity purpose and Rakesh's share profit is ₹18000 then the total profit is 1
   a) ₹46000 b) ₹ 45500 c) ₹ 44000 d) ₹ 45000

5. If Ix-2I ≥ 7, x ∈ R then 1
   a) x ∈ [-5, 9] b) x∈ (-5, 9]
   c) x ∈ $(-\infty, -5) U [9, \infty)$ d) x ∈ $(-\infty, -5) U (9, 8)$

6. The length of a rectangle is double the breadth. If the minimum perimeter of the rectangle is 120 cm, then 1
   a) Breadth $> 20cm$ b) breadth< 20 c) breadth≥ 20 d) Breadth ≤ 20

7. If $\begin{vmatrix} x+1 & -2 \\ x & 2x-3 \end{vmatrix} = 3$, then the integral value(s) of x 1
   a) -2, $\frac{3}{2}$ b) $\frac{1}{2}$ c) -2 d) 3

8. If A is a square matrix, A' is its transpose then $\frac{1}{2}$ (A + A') is 1
   a) A symmetric matrix b) A skew symmetric matrix
   c) A unit matrix d) An elementary matrix

9. If A and B are square matrices of same order, then AB' – BA' is a 1
   a) Skew symmetric matrix b) Symmetric matrix
   c) Null matrix d) unit matrix

10. If A is anon singular matrix, then 1
   a) IAI ≠ IA'I b) $IA^{-1}I \neq A^{-1}$ c) $IAA'I = IA^2I$ d) IAI+IA'I ≠ 0

11. Let A be square matrix of order 3x3 write the value of I 5A I, where IAI = 4 1
   a) 8 b) 500 c) 400 d) 256

12. The maximum value of $\frac{logx}{x}$ is 1
   a) e b) 2e c) $\frac{1}{e}$ d) $\frac{2}{e}$

13. If x is a real, the minimum value of x 1

14. If $\int xe^{kx^2} dx = \frac{1}{4}e^{2x^2} + C$ 1
   a) 4 b) -2 c) 2 d) 1

15. $\int \frac{e^{6logx} - e^{5logx}}{e^{4logx} - e^{3logx}} dx$ 1
   a) $\frac{1}{3}x^3$ +C b) $\frac{1}{5}x^5$ +C c) $\frac{1}{6}x^6$ +C d) $\frac{3}{5}x^5$ +C

16. In a 1000 m race, A reaches the final point in 56 seconds and B reaches in 70 seconds By how much distance A beats B? 1
   a) 100m b) 120 m c) 150m d) 200m

17. If $\int_0^a 3x^2 dx = 8$ then value of a is 1
   a) -2 b) 2 c) 3 d) -3

18. The least value of the function f(x) = ax+ $\frac{b}{x}$ ($x > 0, b > 0, a > 0$) is 1
   a) $\sqrt{ab}$ b) $2\sqrt{ab}$ c) ab d) 2ab

19. The function $f(x)=x^x$, x> 0,has a stationary point at 1
   a) x=e b) x=$\frac{1}{e}$ c) x=1 d) x=$\sqrt{e}$

20. A boat running downstream covers a distance of 16km in two hours while for covering the same distance upstream it takes 4 hours .What is the speed of boat in still water? 1
   a) 4km/h b) 6km/h c) 8km/h d) 10km/h

Section B

21. Two pipes A and B can fill the tank in 24 minutes and 36 minutes respectively. If both the pipes are opened together, the find the time in which the tank will be filled completely. 2

22. A motorboat can row at the speed of 8km/h in still water, If the river is flowing at 4km/h and it takes 16 hours for around trip; find the distance between two points. 2

23. Solve the following system of equations by Cramer's rule: 2
   2x+3y=10, x+6y=4

24. If A=$\begin{bmatrix}3 & 1 & 2\\ 4 & 5 & 3\\ 2 & 4 & 6\end{bmatrix}$, Show that A $+ A^T$ is asymmetric matrix. Where A^T *is the* transpose of the matrix. 2

25. If y = $\log_a x$ then find $\frac{d^2y}{dx^2}$ 2

Section C

26. Given the total cost function for x units of a commodity as 3
C(x) = $\frac{1}{3}x^3 + x^2 - 16x + 2$. Find
(i) The marginal cost function
(ii) The average cost function.

27. Evaluate: $\int \frac{x+e^{2x}}{x^2+e^{2x}}\,dx$ 3

28. Find the intervals in which the following function f(x)= $2x^3-3x^2 -36x +7$ is strictly increasing or strictly decreasing 3

29. Express the matrix A=$\begin{bmatrix}3 & -2 & -4\\ 3 & -2 & -5\\ -1 & 1 & 2\end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix. 3

30. Using properties of determinants prove that $\begin{vmatrix}a+b+2c & a & b\\ c & b+c+2a & b\\ c & a & c+a+2b\end{vmatrix}$ = 2 (a+b + c)^3 3

31. If A= $\begin{bmatrix}-1 & 2\\ 3 & 1\end{bmatrix}$, find f(A), where f(x) = x^2- 2x + 3 3

Section D

32. Evaluate: $\int \frac{1}{x^2(x^4+1)^{3/4}}\,dx$ 5

33. Using matrix method , solve the following system of equations: 5
X - 2y = 10
2x + y + 3z = 8
-2y +z = 7

34. Prove that the area of a right angled triangle of given hypotenuse is maximum when the triangle is isosceles. 5

35. Using properties of determinants prove that: 5
$\begin{vmatrix}a & b-c & c-b\\ a-c & b & c-a\\ a-b & b-a & c\end{vmatrix}$ = (a+ b – c) (b + c - a) (c +a - b)

Or

$\begin{vmatrix}x & y & z\\ x^2 & y^2 & z^2\\ x^3 & y^3 & z^3\end{vmatrix}$ = xyz (x – y) (y - z) (z - x)

Section E

36. Susy is rowing a boat. She takes 6hours to row 48 km upstream whereas she takes 3 hours to go the same distance downstream. Based on the information , answer the following questions:

(i) What is her speed of rowing in still water? 1
(a) 8km/h (b) 12km/h (c) 10km/h (d) 16km/h

(ii) What is the speed of stream? 1
(a) 6km/h (b) 4km/h (c) 8km/h (d) 12km/h

(iii) What is her average speed? 2

37. $P(x) = -5x^2 + 125x + 37500$ is the total profit function of a company, where x is the production of the company. 4x1

Based on the above information, answer the following questions:

(i) What will be the production when the profit is maximum?
(a) 37500 (b) 12.5 (c) -12.5 (d) -37500

(ii) What will be the maximum profit?
(a) Rs.3828125 (b)Rs.38281.25 (c) Rs.39000 (d) Rs.390

(iii) Check in which interval the profit is strictly increasing.
(a) (12.5,∞) (b) For all real numbers
(c) For all positive real numbers (d) (0, 12.5)

(iv) When the production is two units what will be the profit of the company?
(a) 37500 (b) 37730 (c) 37770 (d) none of these

38. Mr. Amar is an architect. He designed a building and provided an entry door in the shape of a rectangle surmounted by a semicircular opening. The perimeter of the door is 10m. 4x1

Based on this information, answer the following questions

(i) If 2x meters and y meters be the breadth and the height of the rectangular part of the door, then the relation between x and y is
(a) $y = 5 - \frac{1}{2}(\pi + 2)x$ (b) $y = 10 - \frac{1}{2}(\pi + 2)x$
(c) $y = 5 - x - \pi x$ (d) $y = 10 - (\pi + 2)x$

(ii) If A (sq.m) is the area enclosed by the door, then
(a) $y = 10x - (\pi + 2)x^2$ (b) $y = 10x - -\frac{1}{2}(\pi + 2)x^2$
(c) $y = 10x - \frac{1}{2}(\pi + 4)x^2$ (d) $y = 10x + \frac{1}{2}(\pi - 4)x^2$

(iii) To allow maximum airflow inside the building, the width of the door
(a) $\frac{10}{\pi+4}$ m (b) $\frac{20}{4+\pi}$ m (c) $\frac{20}{2+\pi}$ m (d) $\frac{40}{2+\pi}$ m

(iv) To allow maximum airflow inside the building ,the height of the door is
(a) $\frac{5}{4+\pi}$ m (b) $\frac{10}{4+\pi}$ m (c) $\frac{20}{4+\pi}$ m (d) $\frac{30}{4+\pi}$ m

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